Graphs of derivatives

First Book


```
GT := 3
```

Introduction

This file provides graphs showing some of the derivatives of many functions you may meet in learning college math. If you own Mathematica 10, you can experiment with this file by modifying it and adding other functions. If you don' t own Mathematica 10 but you own `CDF Reader` (which can be downloaded for free) you can still read it and operate the manipulable graphs.

There is also a Second Book which will appear soon, and eventually a Third Book which will appear when I get around to it.

```
Hyperlink["CDF Reader", "https://www.wolfram.com/cdf-player/"]
```

```
CDF Reader
```

Definition of Derivative

A real valued function f may have a derivative, which is another real valued function denoted by f'. At a point a, f'[a] is the slope of the tangent line to the curve y = f[x] at the point (a, f[x]). This is a precise definition of the derivative of f, but doesn't tell you how to calculate it. That's what you learn in first-semester calculus.

Functions used in creating the examples

```
ShowDerivatives[f_, n_, a_, b_] :=
 Plot[Take[{f[x], f'[x], f''[x], f'''[x], f''''[x]}, n] // Evaluate, {x, a, b},
  Prolog → AbsoluteThickness[GT], PlotRange -> {a, b}, AspectRatio -> 1,
  ImageSize → {200, 200}, PlotStyle -> {{RGBColor[0, 0, 1]}, {RGBColor[1, 0, 0]},
    {RGBColor[0, 1, 0]}, RGBColor[.7, .7, 0], RGBColor[.7, 0, .7]}]
```

TO DO : Make SD print each line in the appropriate color.

TO DO : Increase the type size in the SD printout. It is not obvious how to do this.

TO DO : Bring the graph of the function to the front so that the derivatives all pass underneath it.

```
SD[f_, n_, a_, b_, c_, d_, s_] := ((r := (d - c) / (b - a));
  Plot[Take[{f[x], f'[x], f''[x], f'''[x], f''''[x], f'''''[x]}, n] // Evaluate,
   {x, a, b}, PlotRange -> {c, d}, Prolog → AbsoluteThickness[GT],
   AspectRatio → r, ImageSize → { s, r s},
   PlotStyle -> {{RGBColor[0, 0, 1]}, {RGBColor[1, 0, 0]}, {RGBColor[0, 1, 0]},
     RGBColor[.7, .7, 0], RGBColor[.7, 0, .7], RGBColor[0, .7, .7]}])
```

The derivatives are shown in each graph by color.

function: blue

first derivative : red

second : green

third : gold

fourth : purple

fifth: light blue

DF prints out the formulas for the derivatives

```
DF[f_, n_] :=
 TraditionalForm[
  TableForm[
   Take[{{"function", f[x]}, {"1st deriv", f'[x]},
     {"2nd deriv", f''[x]}, {"3rd deriv", f'''[x]}, {"4th deriv", f''''[x]},
     {"5th deriv", f''''[x]}, {"6th deriv", f''''[x]}}, n]]]
```

Constant function

```
const[x_] := 1
```

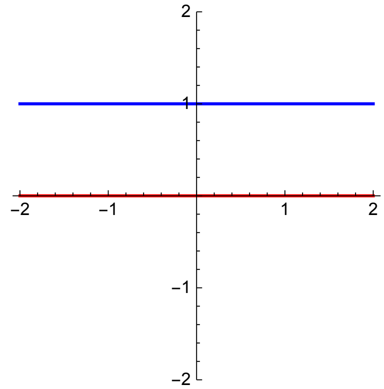
The tangent line to the straight line y = 1 is itself.

```
const'[x]
```

0

The derivative is the constant 0 because the straight line y = 1 has slope 0. You need to know only the definition of derivative to know that. You don' t need to know any "formula".

```
ShowDerivatives[const, 2, -2, 2]
```



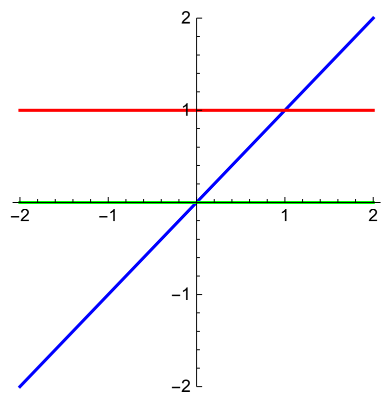
Straight lines

identity function

```
id[x_] := x
```

The identity function has slope = 1 everywhere

```
ShowDerivatives[id, 3, -2, 2]
```

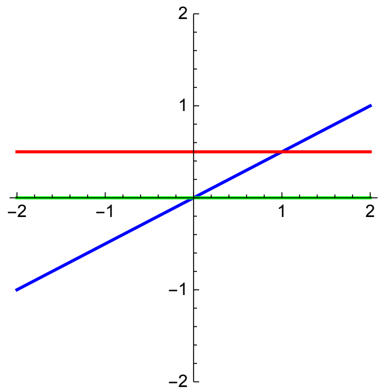



```
(*{{a,0.7},-2,2,.001,Appearance→"Open"},
 {{b,1.2},-2,2,.001,Appearance→"Open"},SaveDefinitions→True]*)
```

straight lines with various slopes

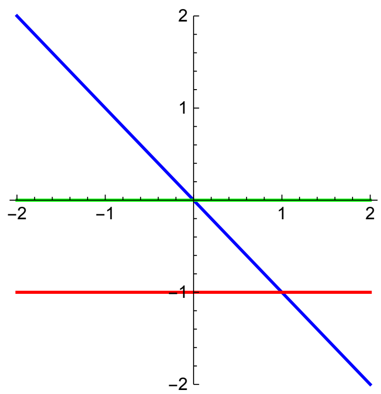
```
slhalf[x_] := 1/2 x
```

```
ShowDerivatives[slhalf, 3, -2, 2]
```



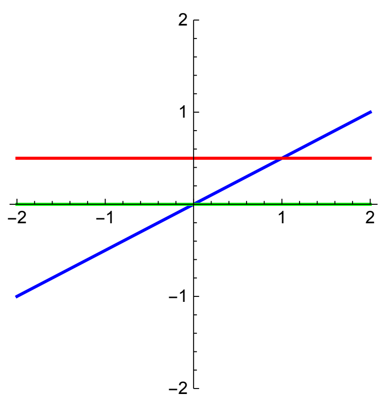

```
sldown[x_] := -x
```

```
ShowDerivatives[sldown, 3, -2, 2]
```



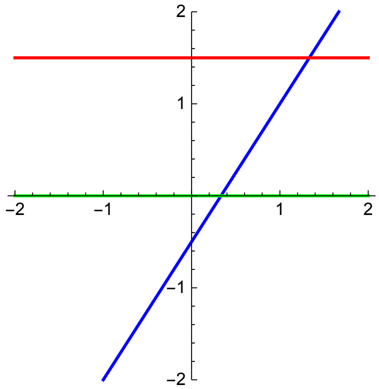

```
slhalf[x_] := 1/2 x
```

```
ShowDerivatives[slhalf, 3, -2, 2]
```



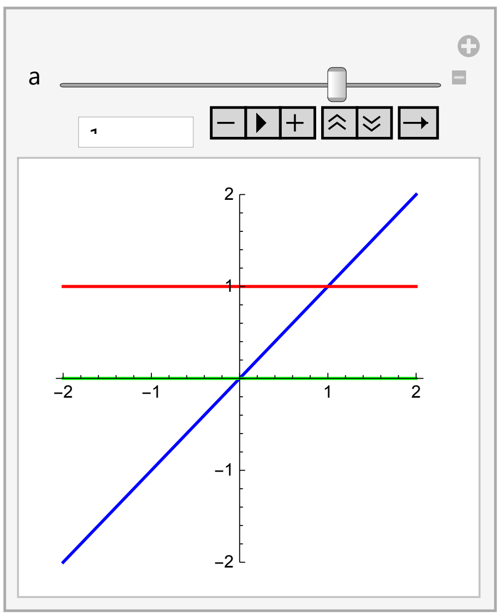

```
slhup[x_] := 1.5 x - .5
```

```
ShowDerivatives[slhup, 3, -2, 2]
```



Manipulable diagram for straight lines

```
Manipulate[
 ShowDerivatives[a * # & , 3, -2, 2], {{a, 1}, -1.99, 1.99, Appearance → "Open"},
 SaveDefinitions → True]
```



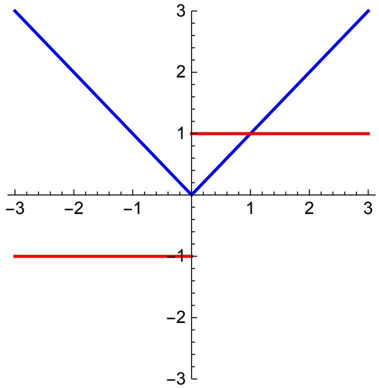
■ Absolute value

There is no tangent line at (0, 0) because the graph has a corner there.

```
leftd[x_] := 1 /; x > 0
```

```
rightd[x_] := -1 /; x < 0
```

```
Plot[{Abs[x], leftd[x], rightd[x]}, {x, -3, 3}, PlotRange → {-3, 3},
 AspectRatio → 1, ImageSize → {200, 200}, Prolog → AbsoluteThickness[GT],
 PlotStyle → {{RGBColor[0, 0, 1]}, {RGBColor[1, 0, 0]},
   {RGBColor[1, 0, 0]}, RGBColor[.7, .7, 0], RGBColor[0.7, 0, .7]}]
```



■ Quadratic function

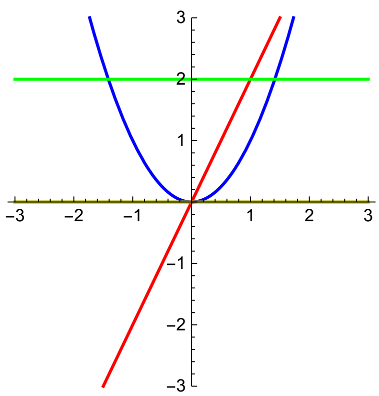
```
Remove[f, g, h]

q1[x_] := x^2

DF[q1, 4]
```

function	x^2
1st deriv	$2x$
2nd deriv	2
3rd deriv	0

```
ShowDerivatives[q1, 4, -3, 3]
```

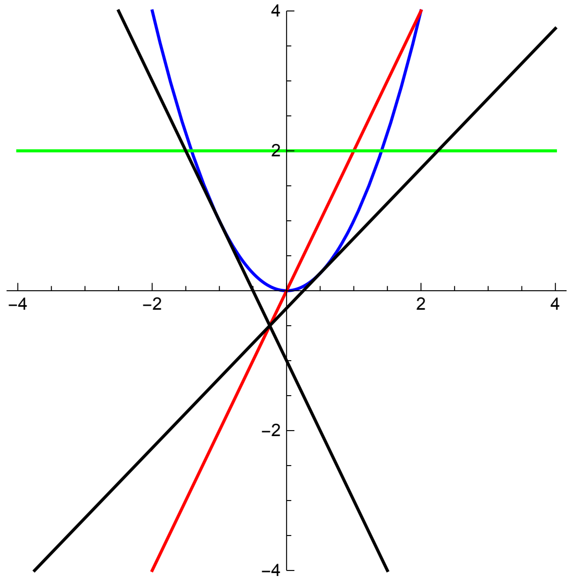


The following graph shows the same graph and two tangent lines (black)

```
g[x_] := x - (1/4)(*Tangent line at (-1,1) with slope -2*)

h[x_] := -2 x - 1(*Tangent line at (.5,.25) with slope -1*)
```

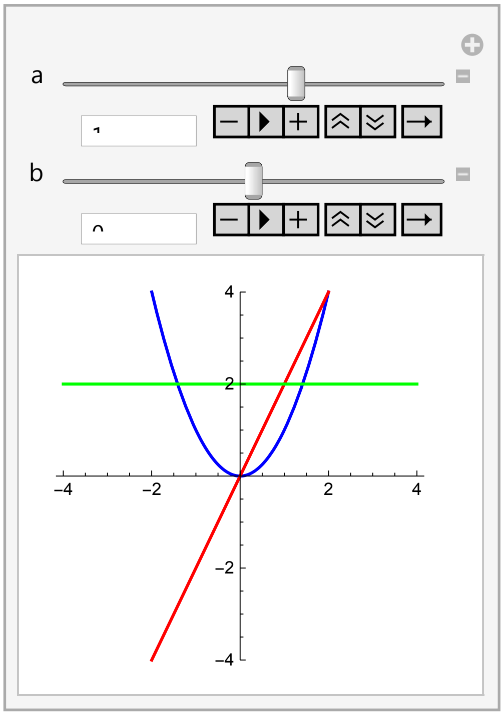
```
Plot[{q1[x], q1'[x], q1''[x], h[x], g[x]}, {x, -4, 4},
 PlotRange → {-4, 4}, AspectRatio → 1, ImageSize → {300, 300},
 Prolog → AbsoluteThickness[GT], PlotStyle → {{RGBColor[0, 0, 1]},
   {RGBColor[1, 0, 0]}, {RGBColor[0, 1, 0]}, RGBColor[0, 0, 0], RGBColor[0, 0, 0]}]
```



Manipulable the coefficients

TO DO : Show tangent lines as well

```
Manipulate[
 ShowDerivatives[a * #^2 + b & , 3, -4, 4], {{a, 1}, -4, 4, Appearance → "Open"},
 {{b, 0}, -3, 3, Appearance → "Open"},
 SaveDefinitions → True]
```

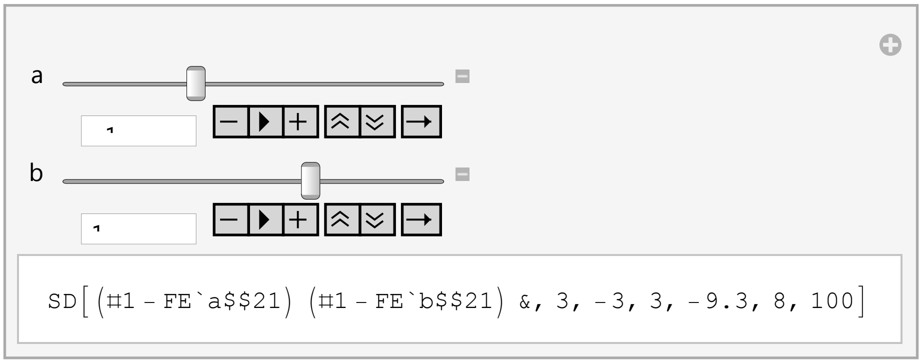


Manipulate the roots

```
DF[(# - a) (# + b) &, 3]
```

function	$(x - a)(b + x)$
1st deriv	$-a + b + 2x$
2nd deriv	2

```
Manipulate[
 SD[(# - a) (# - b) & , 3, -3, 3, -9.3, 8, 100], {{a, -1}, -3, 3, Appearance → "Open"},
 {{b, 1}, -3, 3, Appearance → "Open"},
 SaveDefinitions → True]
```



Cubics

```
Remove[f, g, h]
```

First cubic function

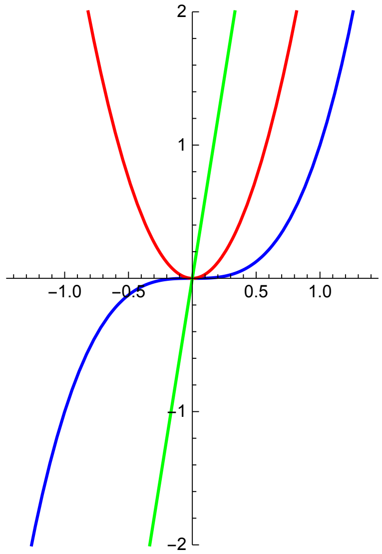
This function has no local maxima or minima and one inflection point.

```
cu1[x_] := x^3
```

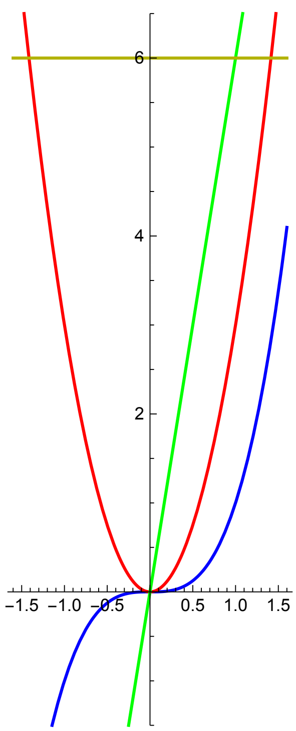
```
DF[cu1, 5]
```

function	x^3
1st deriv	$3 x^2$
2nd deriv	$6 x$
3rd deriv	6
4th deriv	0

```
SD[cu1, 3, -1.4, 1.4, -2, 2, 200]
```




```
SD[cu1, 4, -1.6, 1.6, -1.5, 6.5, 150]
```



Second cubic function

This function has no maxima or minima or critical points. It has one inflection point. It has one real and two complex roots.

cu2[x_] := $x^3 + x$

```
Factor[cu2[x]] // TraditionalForm
```

$x(x^2+1)$

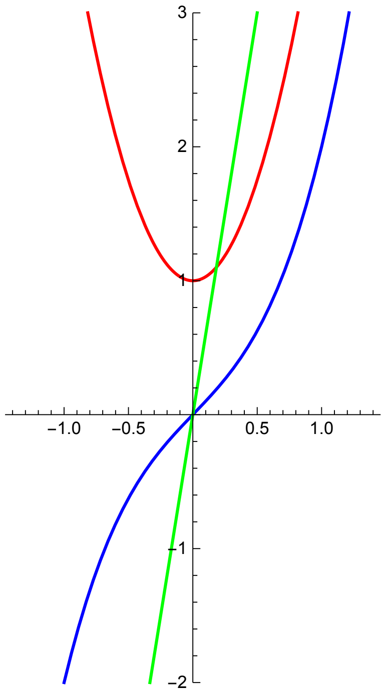
```
Solve[cu2[x] == 0, x]
```

{{x → 0}, {x → -i}, {x → i}}

```
DF[cu2, 3]
```

function	x^3+x
1st deriv	$3x^2+1$
2nd deriv	$6x$

```
SD[cu2, 3, -1.4, 1.4, -2, 3, 200]
```



Third cubic function

This function has no maxima or minima or critical points. It has one inflection point. It has two real roots, one of multiplicity 2.

```
cu3[x_] := x^3 + x^2
```

```
Factor[cu3[x]] // TraditionalForm
```

$x^2(x+1)$

$\partial_x (x^2 (1+x))$

$x^2 + 2x(1+x)$

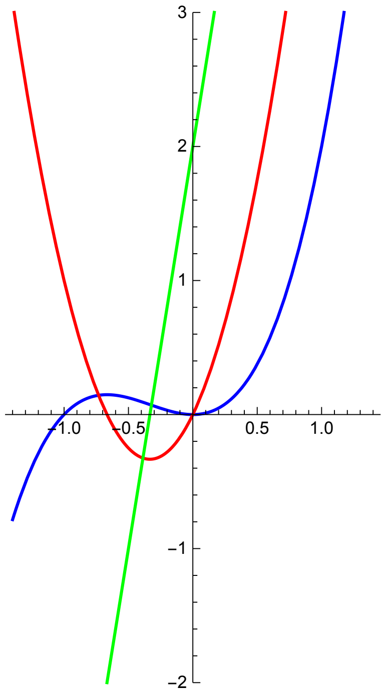
```
Solve[cu3[x] == 0, x]
```

```
{{x → -1}, {x → 0}, {x → 0}}
```

```
DF[cu3, 3]
```

function	$x^3 + x^2$
1st deriv	$3x^2 + 2x$
2nd deriv	$6x + 2$

```
SD[cu3, 3, -1.4, 1.4, -2, 3, 200]
```



Fourth cubic function

This function has local maxima and minima and three distinct real roots.

```
cu4[x_] := x^3 - x
```

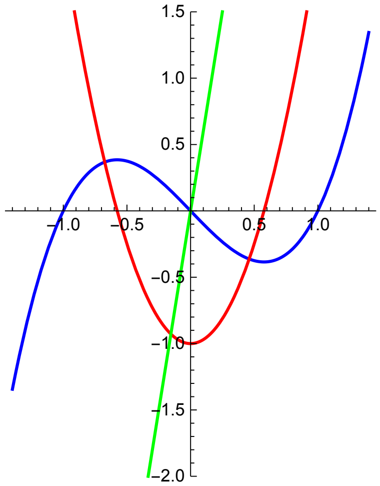
```
Factor[cu4[x]] // TraditionalForm
```

$(x-1)\,x\,(x+1)$

```
DF[cu4, 3]
```

function	$x^3 - x$
1st deriv	$3x^2 - 1$
2nd deriv	$6x$

```
SD[cu4, 3, -1.4, 1.4, -2, 1.5, 200]
```



Fifth cubic function

This function has local maxima and minima and two real roots, one of multiplicity 2.

```
cu5[x_] := x^3 - x^2
```

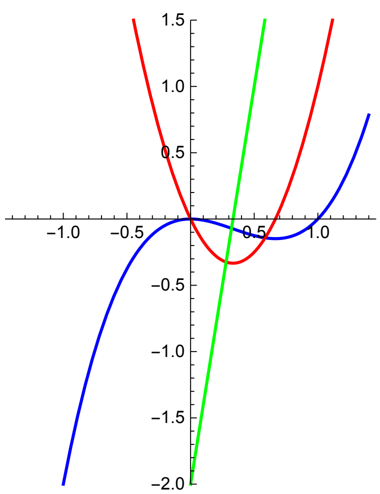
```
Factor[cu5[x]] // TraditionalForm
```

$(x-1)\,x^2$

```
DF[cu5, 3]
```

function	$x^3 - x^2$
1st deriv	$3\,x^2 - 2\,x$
2nd deriv	$6\,x - 2$

```
SD[cu5, 3, -1.4, 1.4, -2, 1.5, 200]
```



Sixth cubic function

This function has no local maxima or minima and three distinct roots, two of them complex.

```
cu6[x_] := x^3 + x^2 + x
```

```
Factor[cu6[x]] // TraditionalForm
```

$x(x^2 + x + 1)$

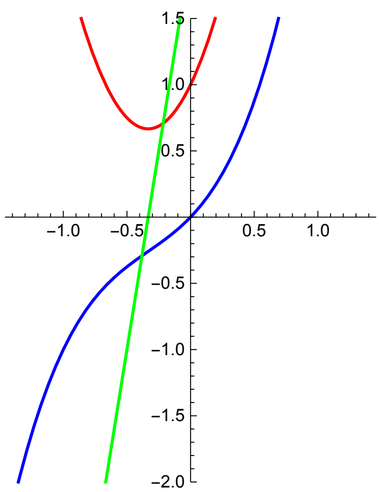
```
Solve[cu6[x] == 0, x] // N
```

```
{{x → 0.}, {x → -0.5 - 0.866025 ⅈ}, {x → -0.5 + 0.866025 ⅈ}}
```

```
DF[cu6, 3]
```

function	$x^3 + x^2 + x$
1st deriv	$3x^2 + 2x + 1$
2nd deriv	$6x + 2$

```
SD[cu6, 3, -1.4, 1.4, -2, 1.5, 200]
```

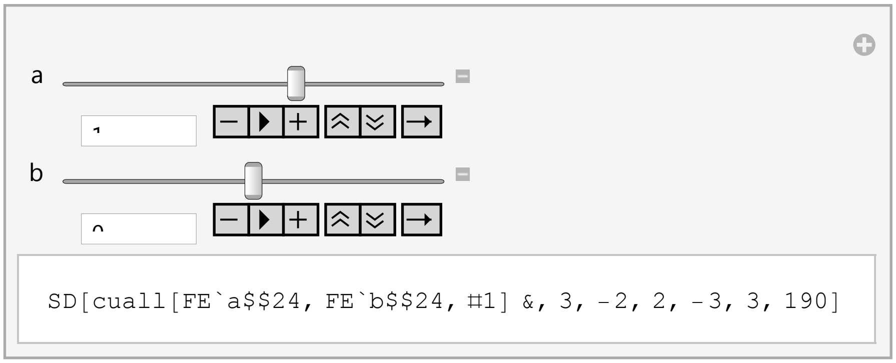


Manipulable diagram of cubic functions

TO DO : This is not as general as it could be

```
cuall[a_, b_, x_] := x^3 + a x^2 + b x
```

```
Manipulate[
 SD[cuall[a, b, #] &, 3, -2, 2, -3, 3, 190], {{a, 1}, -4, 4, Appearance → "Open"},
 {{b, 0}, -3, 3, Appearance → "Open"},
 SaveDefinitions → True]
```



Quartics

```
(.1)^4
```

0.0001

First quartic function

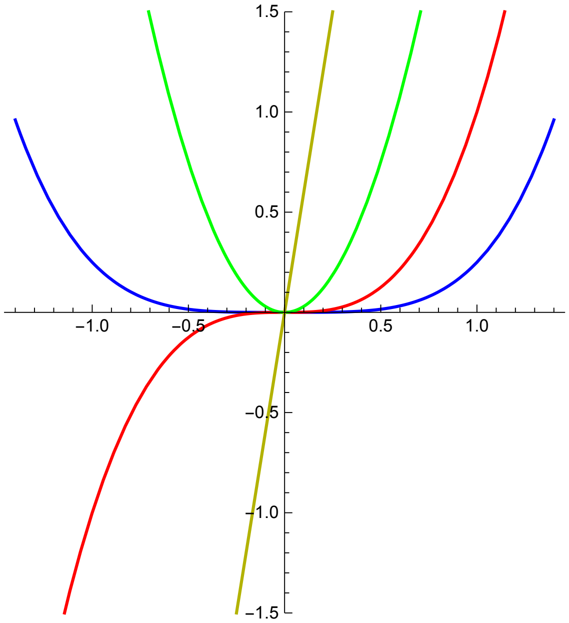
```
quart1[x_] := (1/4) x^4
```

I scaled it by 1/4 because it is hard to see the third derivative for f[x]:=x^4. This function has one quadruple zero at (0,0) and it is *extremely* flat there. Note that (.1)^4 is just .0001. Each of the three derivatives shown has just one zero, at 0, which makes it hard for the function to get off the ground near 0, so to speak.

```
DF[quart1, 4]
```

function	$\frac{x^4}{4}$
1st deriv	x^3
2nd deriv	$3x^2$
3rd deriv	$6x$

```
SD[quart1, 4, -1.4, 1.4, -1.5, 1.5, 300]
```



Second quartic function

This function has two real roots. It has to have at least two since it factors as x times a cubic, and a cubic always has a real root. You can see them in the answer to the Solve function below.

```
quart2[x_] := (1/4) (x^4 - x^2 + x)
```

```
quart2[x]
```

$$\frac{1}{4}\left(x - x^2 + x^4\right)$$

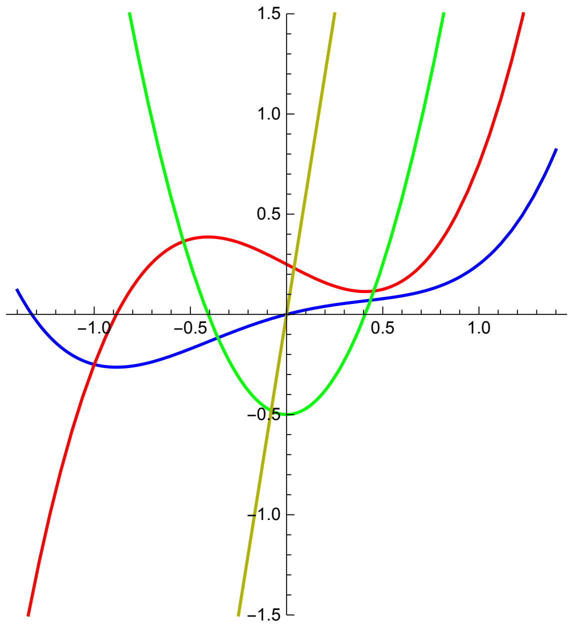
```
Factor[quart2[x]]
```

$$\frac{1}{4} x \left(1 - x + x^3\right)$$

```
Solve[quart2[x] == 0, x] // N
```

{{x → 0.}, {x → -1.32472}, {x → 0.662359 - 0.56228 i}, {x → 0.662359 + 0.56228 i}}

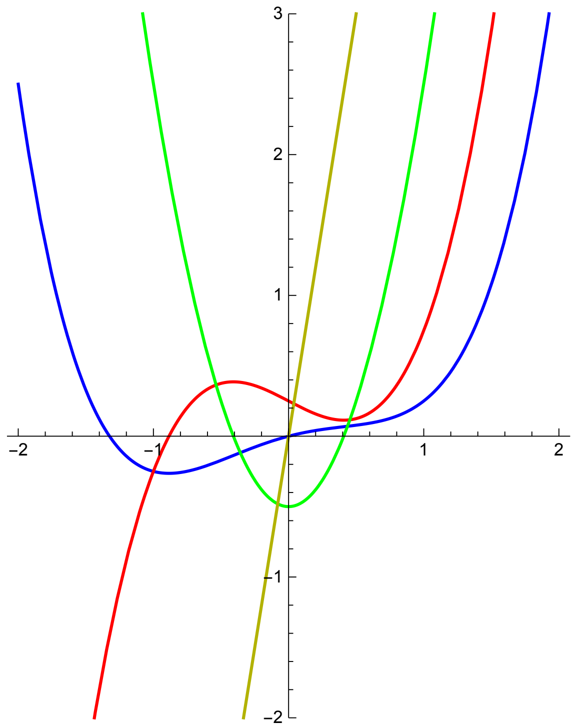
```
SD[quart2, 4, -1.4, 1.4, -1.5, 1.5, 300]
```




```
DF[quart2, 4]
```

function	$\frac{1}{4}\left(x^4 - x^2 + x\right)$
1st deriv	$\frac{1}{4}\left(4\,x^3 - 2\,x + 1\right)$
2nd deriv	$\frac{1}{4}\left(12\,x^2 - 2\right)$
3rd deriv	$6\,x$

```
SD[quart2, 4, -2, 2, -2, 3, 300]
```



Third quartic function

Just like the second quartic function, this function has two real roots. It has to have at least two since it factors as x times a cubic, and a cubic always has a real root. You can see them in the answer to the Solve function below.

```
quart3[x_] := (1/4) (x^4 - x^3 - x)
```

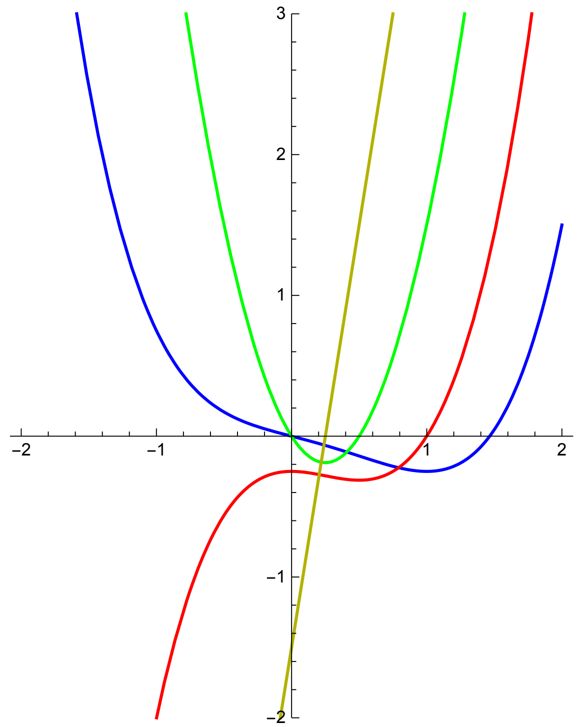
```
Factor[quart3[x]]
```

$$\frac{1}{4} x \left(-1 - x^2 + x^3\right)$$

```
Solve[quart3[x] == 0, x] // N
```

{{x → 0.}, {x → 1.46557}, {x → -0.232786 + 0.792552 i}, {x → -0.232786 - 0.792552 i}}

```
SD[quart3, 4, -2, 2, -2, 3, 300]
```



Fourth quartic function

```
quart4[x_] := (1 / 4) (x^4 - x^3 + x^2 - x)
```

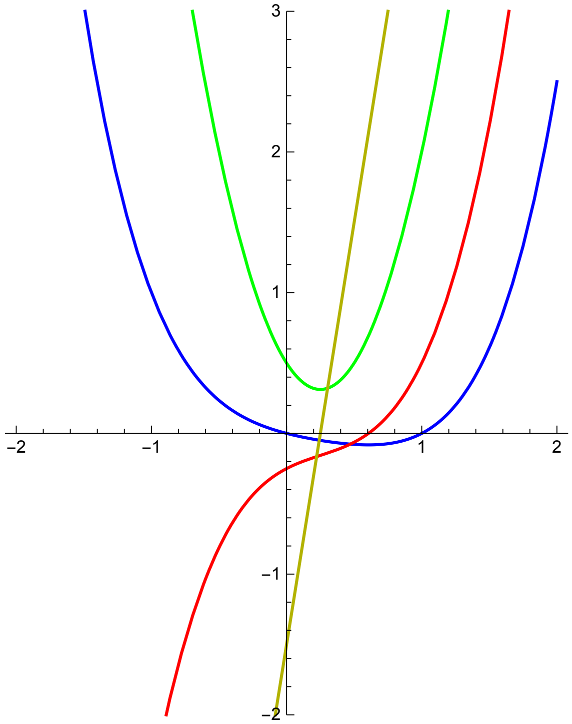
```
Factor[quart4[x]]
```

$\frac{1}{4}(-1+x)\,x\,(1+x^2)$

```
Solve[quart4[x] == 0, x]
```

$\{\{x\to 0\},\{x\to -i\},\{x\to i\},\{x\to 1\}\}$

```
SD[quart4, 4, -2, 2, -2, 3, 300]
```

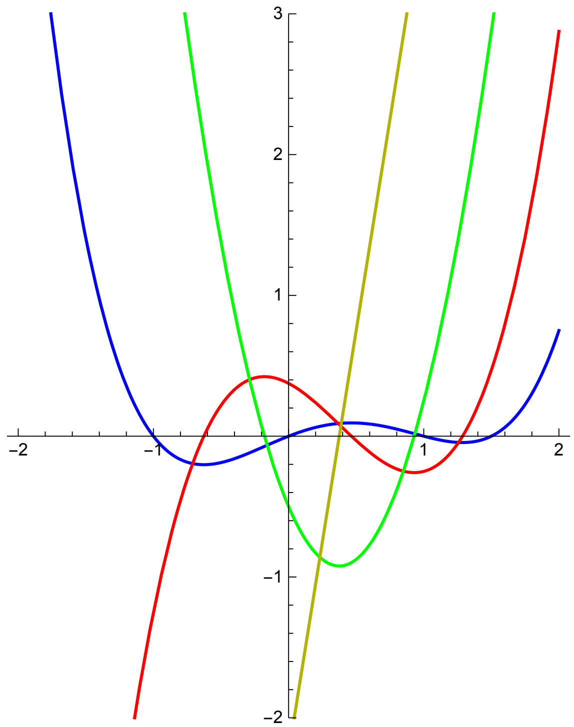


Fifth quartic function

This function has four real roots as you can see from the definition. The derivative has three real roots matching the three inflection points of the function.

```
quart5[x_] := (1 / 4) (x (x + 1) (x - 1) (x - 1.5))
```

```
SD[quart5, 4, -2, 2, -2, 3, 300]
```



Pochhammer

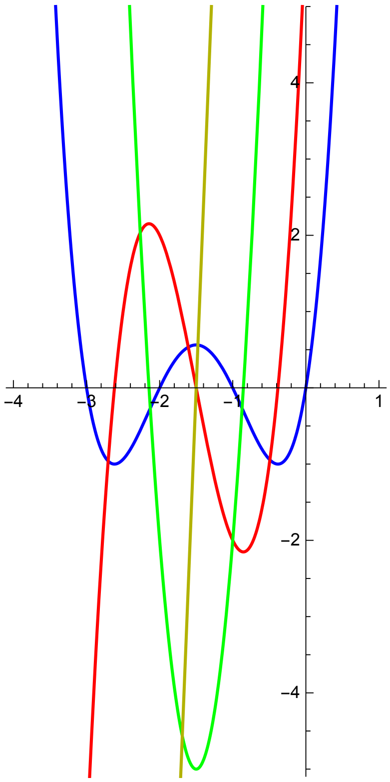
The Pochhammer function of order n is x (x + 1) ... (x + n - 1), so Pochhammer[x, 4] is a quartic.

```
ph4[x_] := Pochhammer[x, 4]
```

```
DF[ph4, 6]
```

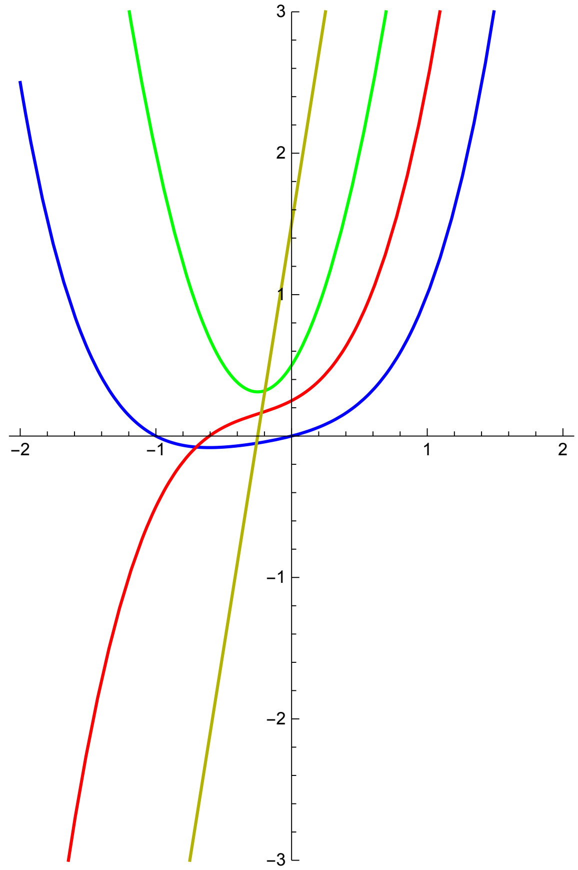
function	$x(x+1)(x+2)(x+3)$
1st deriv	$x(x+1)(x+2)+x(x+3)(x+2)+(x+1)(x+3)(x+2)+x(x+1)(x+3)$
2nd deriv	$2x(x+1)+2(x+2)(x+1)+2(x+3)(x+1)+2x(x+2)+2x(x+3)+2(x+2)(x+3)$
3rd deriv	$6x+6(x+1)+6(x+2)+6(x+3)$
4th deriv	24
5th deriv	24

```
SD[ph4, 5, -4, 1, -5.1, 5, 200]
```

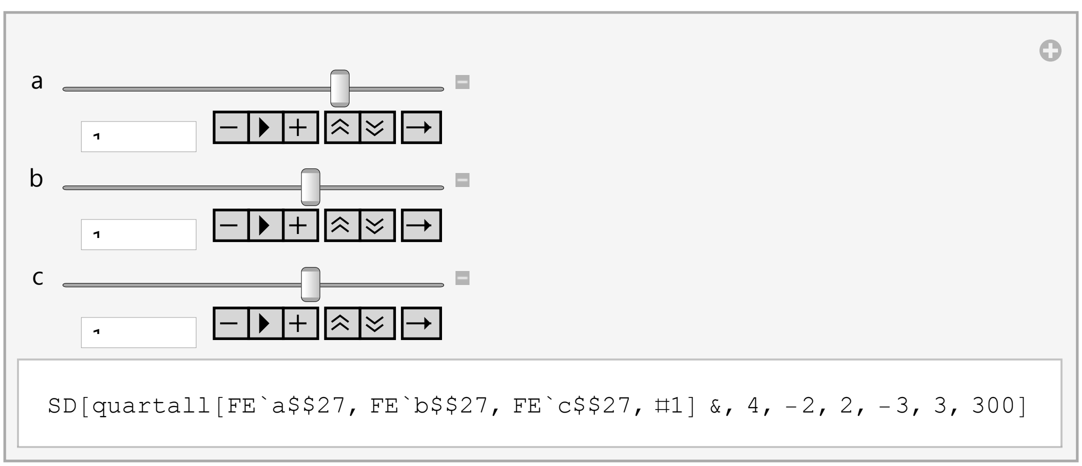



```
quartall[a_, b_, c_, x_] := (1 / 4) (x^4 + a x^3 + b x^2 + c x)
```

```
SD[quartall[1, 1, 1, #] &, 4, -2, 2, -3, 3, 300]
```




```
Manipulate[
 SD[quartall[a, b, c, #] &, 4, -2, 2, -3, 3, 300], {{a, 1}, -2, 2, Appearance → "Open"},
 {{b, 1}, -3, 3, Appearance → "Open"},
 {{c, 1}, -3, 3, Appearance → "Open"},
 SaveDefinitions → True]
```



Quintic

I fiddled with this polynomial until I got the function and all four derivatives to be separated from each other.All the roots of the function and all its derivatives are real and all are shown. Isn' t this gorgeous?

```
quint[x_] := -.5 + .5x +.2 x^2 -.19 x^3 -.015x^4 + .01 x^5
```

```
DF[quint, 5]
```

function	$0.01\,x^5 - 0.015\,x^4 - 0.19\,x^3 + 0.2\,x^2 + 0.5\,x - 0.5$
1st deriv	$0.05\,x^4 - 0.06\,x^3 - 0.57\,x^2 + 0.4\,x + 0.5$
2nd deriv	$0.2\,x^3 - 0.18\,x^2 - 1.14\,x + 0.4$
3rd deriv	$0.6\,x^2 - 0.36\,x - 1.14$
4th deriv	$1.2\,x - 0.36$

```
Solve[quint[x] == 0, x] // N
```

```
{{x → -3.76822}, {x → -1.79188}, {x → 0.984639}, {x → 1.7311}, {x → 4.34436}}
```

```
Solve[quint'[x] == 0, x] // N
```

```
{{x → -3.04805}, {x → -0.674148}, {x → 1.37002}, {x → 3.55217}}
```

```
Solve[quint''[x] == 0, x] // N
```

```
{{x → -2.16289}, {x → 0.339541}, {x → 2.72335}}
```

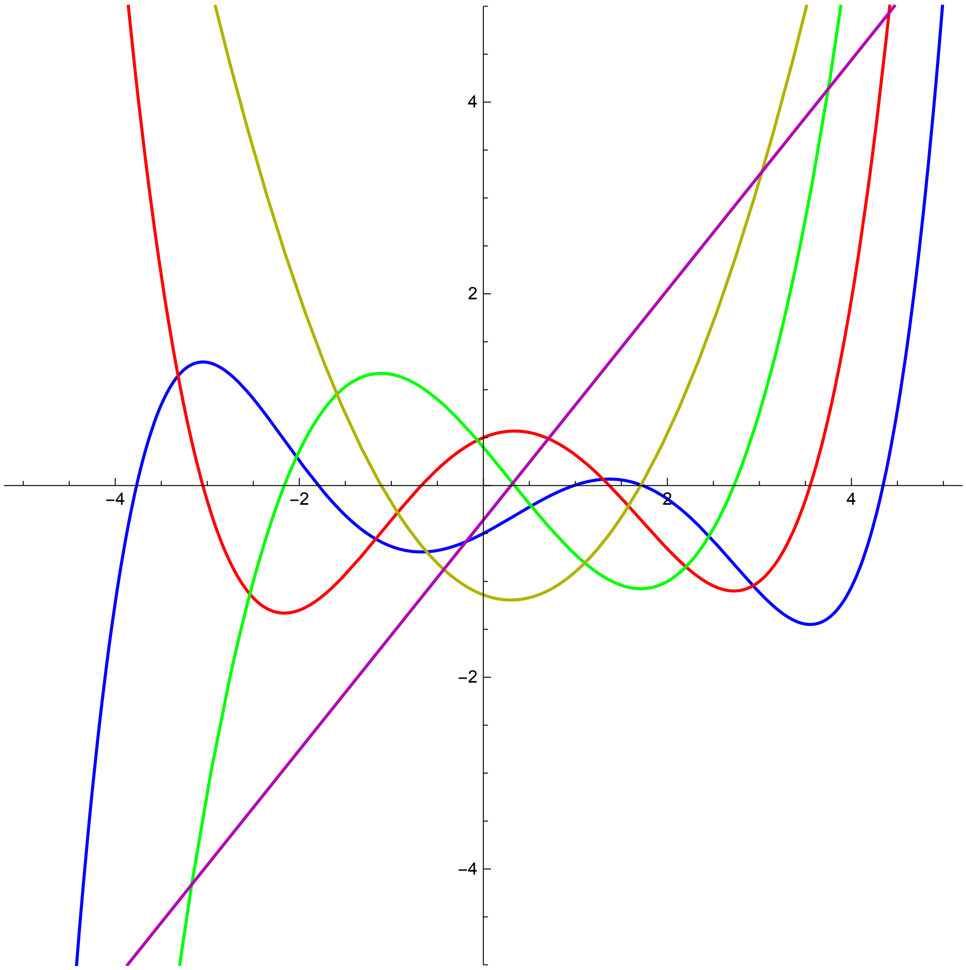
```
Solve[quint'''[x] == 0, x] // N
```

```
{{x → -1.11067}, {x → 1.71067}}
```

```
Solve[quint''''[x] == 0, x] // N
```

```
{{x → 0.3}}
```

```
SD[quint, 5, -5, 5, -5, 5, 500]
```



Rational functions

First rational function

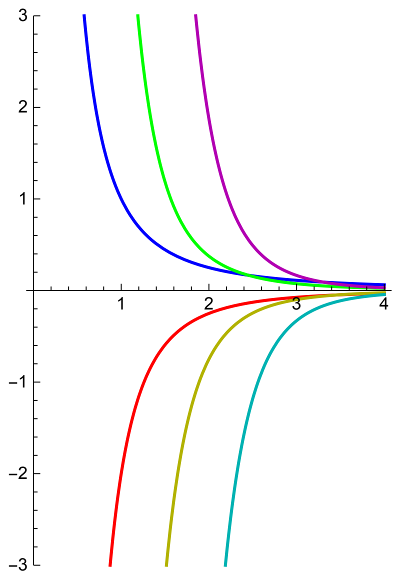
This is the right half of a hyperbola that has a pole at x = 0.

```
rat1[x_] := 1/(x^2)
```

```
DF[rat1, 6]
```

function	$\frac{1}{x^2}$
1st deriv	$-\frac{2}{x^3}$
2nd deriv	$\frac{6}{x^4}$
3rd deriv	$-\frac{24}{x^5}$
4th deriv	$\frac{120}{x^6}$
5th deriv	$\frac{120}{x^6}$

```
SD[rat1, 6, 0.01, 4, -3, 3, 200]
```



Second rational function

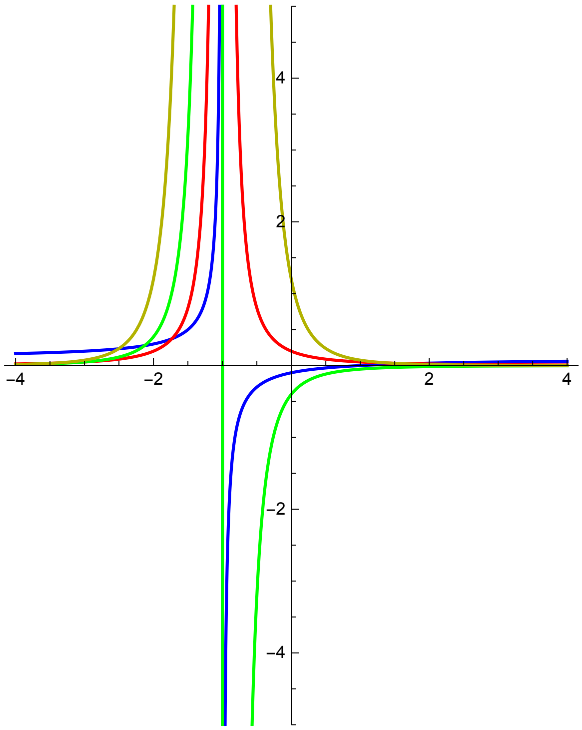
The denominator is 0 at x = -1, so there is a pole there. The asyptote is shown by a vertical green line

```
rat2[x_] := (1/10) (x - 1) / (x + 1)
```

```
DF[rat2, 5]
```

function	$\frac{x-1}{10(x+1)}$
1st deriv	$\frac{1}{10(x+1)} - \frac{x-1}{10(x+1)^2}$
2nd deriv	$\frac{x-1}{5(x+1)^3} - \frac{1}{5(x+1)^2}$
3rd deriv	$\frac{3}{5(x+1)^3} - \frac{3(x-1)}{5(x+1)^4}$
4th deriv	$\frac{12(x-1)}{5(x+1)^5} - \frac{12}{5(x+1)^4}$

```
SD[rat2, 4, -4, 4, -5, 5, 300]
```



Third rational function

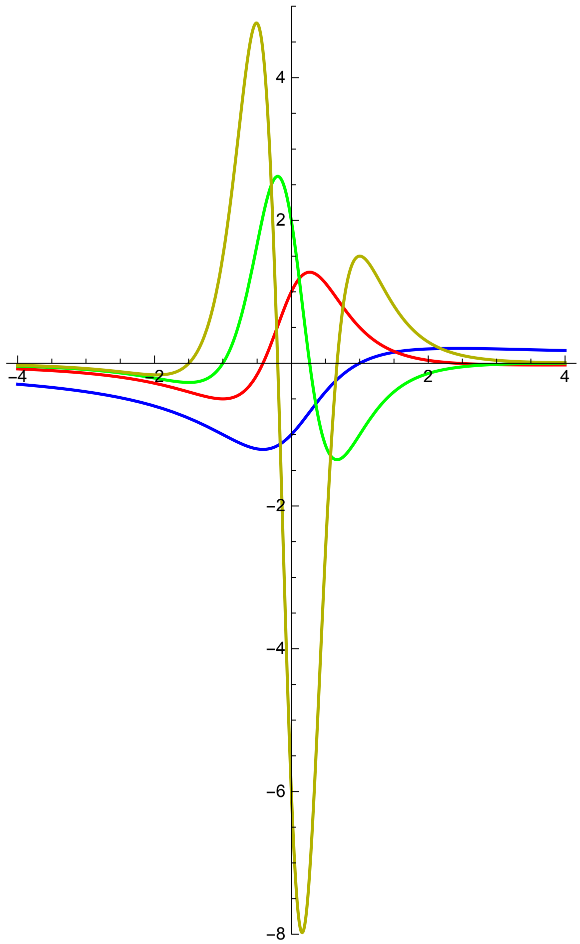
The denominator has no zeros so this is an entire function (which means it has no poles).

```
rat3[x_] := (x - 1) / (x^2 + 1)
```

```
DF[rat3, 5]
```

function	$\frac{x-1}{x^2+1}$
1st deriv	$\frac{1}{x^2+1} - \frac{2(x-1)x}{(x^2+1)^2}$
2nd deriv	$\frac{8(x-1)x^2}{(x^2+1)^3} - \frac{4x}{(x^2+1)^2} - \frac{2(x-1)}{(x^2+1)^2}$
3rd deriv	$\frac{24x^2}{(x^2+1)^3} + \frac{24(x-1)x}{(x^2+1)^3} - \frac{6}{(x^2+1)^2} - \frac{48(x-1)x^3}{(x^2+1)^4}$
4th deriv	$-\frac{288(x-1)x^2}{(x^2+1)^4} + \frac{96x}{(x^2+1)^3} + \frac{24(x-1)}{(x^2+1)^3} + \frac{384(x-1)x^4}{(x^2+1)^5} - \frac{192x^3}{(x^2+1)^4}$

```
SD[rat3, 4, -4, 4, -8, 5, 300]
```



Fourth rational function

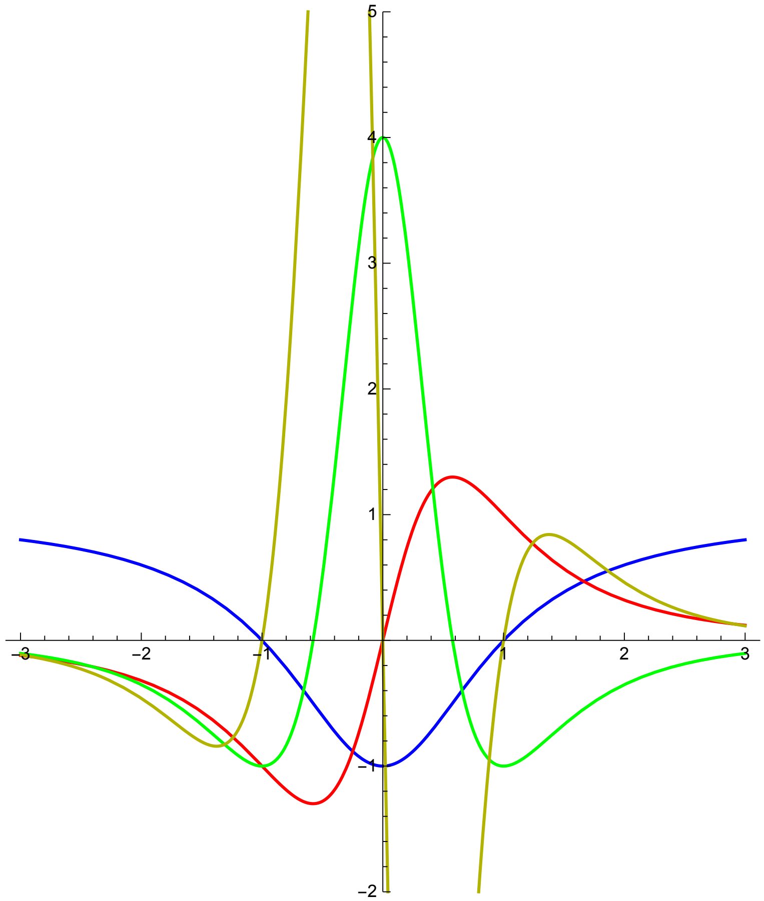
```
rat4[x_] := (x^2 - 1) / (x^2 + 1)
```

```
DF[rat4, 5]
```

function	$\frac{x^2-1}{x^2+1}$
1st deriv	$\frac{2x}{x^2+1} - \frac{2x(x^2-1)}{(x^2+1)^2}$
2nd deriv	$-\frac{8x^2}{(x^2+1)^2} + \frac{8(x^2-1)x^2}{(x^2+1)^3} + \frac{2}{x^2+1} - \frac{2(x^2-1)}{(x^2+1)^2}$
3rd deriv	$-\frac{24x}{(x^2+1)^2} + \frac{24(x^2-1)x}{(x^2+1)^3} + \frac{48x^3}{(x^2+1)^3} - \frac{48(x^2-1)x^3}{(x^2+1)^4}$
4th deriv	$\frac{288x^2}{(x^2+1)^3} - \frac{288(x^2-1)x^2}{(x^2+1)^4} - \frac{24}{(x^2+1)^2} + \frac{24(x^2-1)}{(x^2+1)^3} - \frac{384x^4}{(x^2+1)^4} + \frac{384(x^2-1)x^4}{(x^2+1)^5}$

The third derivative reaches up to about 9.3 and down to about - 9.5

```
SD[rat4, 4, -3, 3, -2, 5, 400]
```



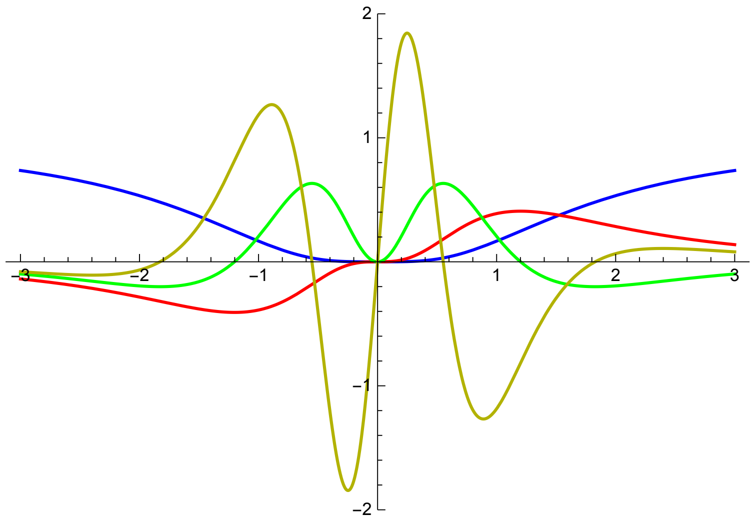
Fifth rational function

```
rat5[x_] := (x^4) / ((x^2 + 1) (x^2 + 2))
```

```
DF[rat5, 6]
```

function	$\frac{x^4}{(x^2+1)(x^2+2)}$
1st deriv	$-\frac{2x^5}{(x^2+1)^2(x^2+2)}-\frac{2x^5}{(x^2+1)(x^2+2)^2}+\frac{4x^3}{(x^2+1)(x^2+2)}$
2nd deriv	$\frac{12x^2}{(x^2+1)(x^2+2)}+\frac{8x^6}{(x^2+1)^3(x^2+2)}+\frac{8x^6}{(x^2+1)^2(x^2+2)^2}+\frac{8x^6}{(x^2+1)(x^2+2)^3}-\frac{18x^4}{(x^2+1)^2(x^2+2)}-\frac{18x^4}{(x^2+1)(x^2+2)^2}$
3rd deriv	$\frac{24x}{(x^2+1)(x^2+2)}-\frac{48x^7}{(x^2+1)^4(x^2+2)}-\frac{48x^7}{(x^2+1)^3(x^2+2)^2}-\frac{48x^7}{(x^2+1)^2(x^2+2)^3}-\frac{48x^7}{(x^2+1)(x^2+2)^4}+\frac{120x^5}{(x^2+1)^3(x^2+2)}+\frac{120x^5}{(x^2+1)^2(x^2+2)^2}+\frac{120x^5}{(x^2+1)(x^2+2)^3}$
4th deriv	$-\frac{336x^2}{(x^2+1)^2(x^2+2)}-\frac{336x^2}{(x^2+1)(x^2+2)^2}+\frac{24}{(x^2+1)(x^2+2)}+\frac{384x^8}{(x^2+1)^5(x^2+2)}+\frac{384x^8}{(x^2+1)^4(x^2+2)^2}+\frac{384x^8}{(x^2+1)^3(x^2+2)^3}+\frac{384x^8}{(x^2+1)^2(x^2+2)^4}+\frac{384x^8}{(x^2+1)(x^2+2)}$
5th deriv	$-\frac{336x^2}{(x^2+1)^2(x^2+2)}-\frac{336x^2}{(x^2+1)(x^2+2)^2}+\frac{24}{(x^2+1)(x^2+2)}+\frac{384x^8}{(x^2+1)^5(x^2+2)}+\frac{384x^8}{(x^2+1)^4(x^2+2)^2}+\frac{384x^8}{(x^2+1)^3(x^2+2)^3}+\frac{384x^8}{(x^2+1)^2(x^2+2)^4}+\frac{384x^8}{(x^2+1)(x^2+2)}$

```
SD[rat5, 4, -3, 3, -2, 2, 400]
```



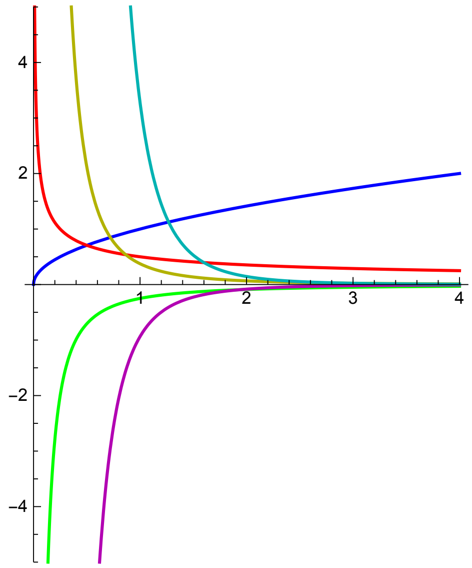
Functions involving roots

Square root

```
DF[Sqrt, 6]
```

function	$\sqrt{x}$
1st deriv	$\frac{1}{2\sqrt{x}}$
2nd deriv	$-\frac{1}{4x^{3/2}}$
3rd deriv	$\frac{3}{8x^{5/2}}$
4th deriv	$-\frac{15}{16x^{7/2}}$
5th deriv	$-\frac{15}{16x^{7/2}}$

```
SD[Sqrt, 6, -4, 4, -5, 5, 240]
```



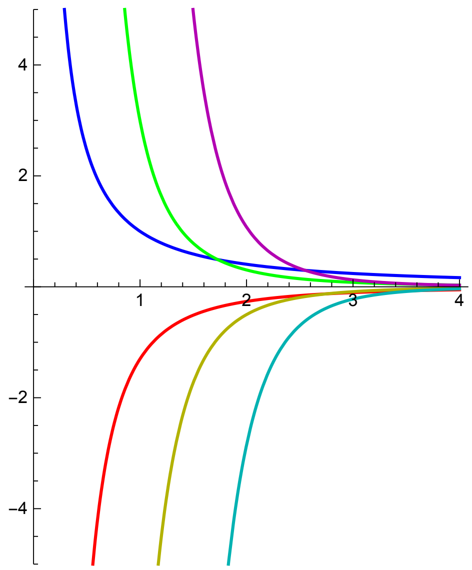
Cube root

```
curt[x_] := 1 / (x^(1.3))
```

```
DF[curt, 6]
```

function	$\frac{1}{x^{1.3}}$
1st deriv	$-\frac{1.3}{x^{2.3}}$
2nd deriv	$\frac{2.99}{x^{3.3}}$
3rd deriv	$-\frac{9.867}{x^{4.3}}$
4th deriv	$\frac{42.4281}{x^{5.3}}$
5th deriv	$\frac{42.4281}{x^{5.3}}$

```
SD[curt, 6, -4, 4, -5, 5, 240]
```



Square root of $x^2 + 1$

```
sqxsq[x_] := 1 / (Sqrt[x^2 + 1])
DF[sqxsq, 6]
```

function	$\frac{1}{\sqrt{x^2+1}}$
1st deriv	$-\frac{x}{(x^2+1)^{3/2}}$
2nd deriv	$\frac{3x^2}{(x^2+1)^{5/2}} - \frac{1}{(x^2+1)^{3/2}}$
3rd deriv	$\frac{9x}{(x^2+1)^{5/2}} - \frac{15x^3}{(x^2+1)^{7/2}}$
4th deriv	$-\frac{90x^2}{(x^2+1)^{7/2}} + \frac{9}{(x^2+1)^{5/2}} + \frac{105x^4}{(x^2+1)^{9/2}}$
5th deriv	$-\frac{90x^2}{(x^2+1)^{7/2}} + \frac{9}{(x^2+1)^{5/2}} + \frac{105x^4}{(x^2+1)^{9/2}}$

```
SD[sqxsq, 4, -4, 4, -2, 2, 400]
```